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A Discourse Concerning Algebra: English Algebra to 1685
By Jacqueline A. Stedall. Oxford (Oxford University Press). 2002. ISBN 0-198-52495-1. 294 pp. $116

The Greate Invention of Algebra: Thomas Harriot's Treatise on Equations
By Jacqueline A. Stedall. Oxford (Oxford University Press). 2003. ISBN 0-198-52602-4. 322 pp. $136

Some years ago, at the Institute for the History of Mathematics and Its Use in Teaching, speakers Helena Pycior and John Fauvel sparked my interest in 17th-century British algebra with their tales of the mysterious Thomas Harriot (1560?–1621) and John Pell (1611–1685), and the prolific and inventive John Wallis (1616–1703). A few years later, during my first visit to England to study manuscripts of 16th- and 17th-century British algebraists, I was guided by a remarkably thorough and insightful series of papers by a scholar who had completed her Ph.D. dissertation with Fauvel, Jacqueline Stedall. Stedall has now written a book on English algebra through 1685 that includes work from these papers and much more. Readers of this review may recognize 1685 as the year in which John Wallis published his *Treatise of Algebra* and indeed Stedall's *A Discourse Concerning Algebra*: *English Algebra to 1685* takes Wallis's treatise as both its topic and its organizing principle. Since Wallis's book was a history as well as a manual of English algebra,[1] this allows Stedall to recount the entire history of English algebra to 1685, analyzing Wallis's account and shedding new light on it from her own research. That Wallis focused on 17th-century English algebra means that Stedall's book also concentrates on the English algebra of that period. The reader thus has the benefit, not only of a leading 17th-century mathematician's perspective on the algebra up to and of his day, but also of a modern scholar's analysis and extension of it.

Wallis's *Treatise of Algebra* contained 100 chapters, the first 14 of which covered the history of algebra from ancient times up to about 1600, with emphasis on English mathematicians and how algebraic ideas entered England. Chapters 15–29 are on the algebra of William Oughtred (1573–1660), Chapters 30–56 on that of Thomas Harriot, Chapters 57–72 deal with the work of John Pell, Chapters 73–97 cover Wallis's own work in his 1656 *Arithmetica Infinitorum* and the work Isaac Newton based on it, Chapters 98 and 99 focus on work of William Brouncker (1620–1684) in number theory, and Chapter 100 serves as the conclusion. Stedall follows Wallis's outline very closely, with Chapters 2 through 7 covering, respectively, algebra up to about 1600, Oughtred, Harriot, Pell, Wallis and Newton, and Brouncker. Stedall claims her book to be "a new contribution, based on Wallis's foundations, to the study of early modern algebra" (p. 17), and indeed it is, with every chapter providing new revelations, or at least clarifications or corrections of past scholarship, about the featured mathematician(s).

In Chapter 2, Stedall focuses on Wallis as historian, a role for which he is less well known, highlighting both the strengths and weaknesses of his historical research. She shows how he used original sources to trace the introduction to England of Indo-Arabic numbers, which he called "Numeral Figures," and argues that in this, unlike in much of his

[1] Its full title was *A Treatise of Algebra, both Historical and Practical. Shewing, The Original, Progress, and Advancement thereof, from time to time*; *and by what Steps it hath attained to the Heighth at which now it is*.

other historical work, Wallis's historical research methods were quite advanced, making him "perhaps the first modern historian of mathematics" (p. 17). She includes a discussion of the Helmdon mantelpiece, which seems to contain the date 1133 written in mixed Roman and Indo-Arabic numerals and which both she and Wallis visited and deciphered. In addition to its relevance to her narrative, the mantelpiece gives Stedall a personal connection to Wallis that authors of books on historical figures do not always enjoy. She has an even stronger personal connection with Wallis via the collection of mathematics books and manuscripts in the Bodleian Library at Oxford University, to which both she and Wallis had ready access in their positions at Oxford and over which both she and Wallis pored in their studies of early English and European mathematics. Stedall reports that Wallis's "annotations are to be found frequently in both the books and manuscripts" (p. 12) of the Savile collection, in particular.

Having given Wallis's and her own history of algebra up to about 1600 in Chapter 2, and in keeping with Wallis's emphasis on his own century, Stedall devotes much of the remainder of her book to English mathematics from 1631 to 1685. In her chapter on William Oughtred, she describes the huge influence of his *Clavis Mathematicae*, first published in 1631, on leading 17th-century mathematicians and scientists, and the book's resulting undue longevity (seven editions over 72 years, plus 15 chapters devoted to it in Wallis's *Treatise of Algebra*). She recounts the roles of Wallis and others in keeping Oughtred's rather old-fashioned and cumbersome text in print for so long, possibly hindering the publication of more advanced modern texts. In this and other chapters, we learn much about the difficulty of publishing mathematics books in 17th-century England, as well as about the few algebra books that did appear during this period.

In her chapter on Thomas Harriot, Stedall compares Harriot's algebra as it appears in his manuscripts with that in his posthumously published *Artis Analyticae Praxis* (1631). She shows that, had the editors of the *Praxis* arranged Harriot's work as he had intended, his understanding of the relationship between coefficients and roots of polynomial equations would have been much more apparent, and that, had they included more of Harriot's work, his willingness and ability to handle negative and complex roots would also have been evident.[2] Intriguingly, Harriot's algebra as presented by Wallis followed the manuscripts rather than the *Praxis*, although Wallis seemed to imply that he had used only the latter as a source. Wallis drew much criticism for what seemed to be his gross exaggeration of the extent of Harriot's work in algebra. Stedall presents new evidence that Wallis obtained his knowledge of Harriot's algebra from John Pell, who was familiar with Harriot's manuscript work, but that Wallis honored Pell's wish to remain anonymous.

So little is known about the mysterious John Pell that practically any new information about him and his mathematics is revelatory, and Stedall provides some especially tantalizing items in her chapters on Harriot and on Pell himself. Taking into account Pell's contributions to the 27 chapters of Wallis's *Treatise of Algebra* devoted to Harriot, she claims that Pell was responsible for "well over one-third" (p. 152) of Wallis's book. Very little of this work, even in the 16 chapters devoted to Pell, was attributed to him, due, Stedall argues, to his strong desire for anonymity. While Pell's contributions to Thomas Brancker's 1668 *An Introduction to Algebra* and to its source, Johann Rahn's *Teutsche Algebra,* are relatively well known, Pell's anonymously published 1638 tract, *An Idea of Mathematics,* remains less familiar. Based on this publication, Stedall claims for Pell a belief in an "inherent logical structure by means of which all mathematical theorems could be deduced" (p. 131), a unique and forward-looking view of mathematics for his day.[3]

Wallis's own development of an infinite product for $4/\pi$ in his 1656 *Arithmetica Infinitorum* and Newton's ensuing development of the (general) binomial theorem in 1664 constitute perhaps the best known work in both Wallis's *Treatise of Algebra* and Stedall's *Discourse Concerning Algebra*. Stedall's improvements over the many mathematicians and historians who have described this material previously include her attempt to approach it with 17th-century sensibility and understanding, her discussion of contemporary reaction to Wallis's work, and her tracing of the influence of Wallis through Newton's work on the binomial theorem. While we might not classify this material as algebra, Stedall makes clear that Wallis viewed it as the "arithmetic of infinite sums" (p. 158) and argues that his work "ensured that the 17th-century trend from geometric to algebraic thinking became irreversible" (p. 155). She also points out that

[2] See my comments on Stedall's *The Greate Invention of Algebra* later in this review.

[3] Together with Noel Malcolm, Stedall has since published a book on Pell's life, work, and correspondence, *John Pell (1611–1685) and His Correspondence with Sir Charles Cavendish*: *The Mental World of an Early Modern Mathematician* (Oxford: Oxford Univ. Press, 2005).

Wallis, especially in his later account in the *Treatise of Algebra*, noted that he had found a new type of number and hinted at the distinction between algebraic and transcendental numbers.[4]

William Brouncker remains even less well known to us than Pell, despite his having been the first president of the Royal Society from 1662 to 1677. Stedall describes his mathematical achievements, including his representation of $4/\pi$ as an infinite continued fraction and his solution of number theoretic problems posed by Pierre Fermat. (Fermat's problems involved a relationship that came to be known as Pell's equation, but that should have been named after Brouncker or, perhaps better, Fermat himself.) Much in this chapter will be new to readers, including Stedall's very plausible reconstruction of Brouncker's continued fraction representation of $4/\pi$ using only 17th-century methods. Again, although we would classify much of Brouncker's work as number theory, Stedall justifies Wallis's inclusion of it in his *Treatise of Algebra* by emphasizing Brouncker's general and very algebraic approach to almost every problem he solved.

In her conclusion, Stedall returns to her analysis of Wallis as a historian, noting that his *Treatise of Algebra*, with its focus on 17th-century English mathematics, was in large part Wallis's own mathematical history. While admitting the weaknesses of such a narrow approach to what was billed as a history of algebra, she contends that the rapid mathematical changes in England during Wallis's lifetime actually led him to become the "first mathematician to present mathematics as a system of living, changing ideas," rather than as "revealed knowledge handed down from one generation to the next" (p. 217).

The only other comprehensive study of the British algebra of this period is Helena Pycior's *Symbols, Impossible Numbers, and Geometric Entanglements*: *British Algebra through the Commentaries on Newton's Universal Arithmetick* (Cambridge: Cambridge Univ. Press, 1997), and one should read both books in order to obtain a more complete picture. While both Pycior and Stedall trace English mathematicians' increasing acceptance and use of algebra, Pycior is much more concerned with the status of algebra as a mathematical discipline among English mathematicians, especially as compared with that of classical geometry. Stedall discusses Wallis's and Newton's (differing) views on the respective roles of algebra and geometry, but concludes that their concerns "came rather too late, for by the later years of the 17th-century Wallis and Newton had both played major roles in ensuring that every aspect of mathematics, not just geometry, had come to be handled algebraically" (p. 214).

Full of exciting new discoveries, insights, and interpretations and engagingly written, Jacqueline Stedall's *A Discourse Concerning Algebra* is an important contribution to the history of algebra and an immense pleasure to read. All historians of mathematics, including specialists in the history of algebra and/or 17th-century British mathematics, are sure to learn something new from every chapter of the book and to enjoy Stedall's compelling account of the rise of algebra and of algebraic thinking in 17th-century England.

In *The Greate Invention of Algebra*: *Thomas Harriot's Treatise on Equations*, Stedall presents Thomas Harriot's algebra (his theory of polynomial equations and their roots) as it appears in his manuscripts, rather than in the posthumously published *Artis Analyticae Praxis* (1631). Harriot's manuscript work is both more coherent and more complete than the *Praxis*, and includes consideration of negative and complex roots omitted from the published book. Although the surviving manuscript sheets are now quite out of order, Stedall has reassembled 140 of them using Harriot's pagination and the notes of his friend and contemporary Nathaniel Torporley (1564–1632) as her guides to how Harriot's algebra *should* have been arranged by the editors of the published *Praxis*.

Stedall's brief but excellent introduction provides relevant information about Harriot's life and work, including his study of Viète's algebra, his innovations in algebraic notation, his breakthroughs in the understanding of roots of equations in terms of their coefficients, the posthumous publication of some of his algebra in the *Praxis*, Torporley's criticism of the *Praxis*, and Harriot's influence on the next generation of mathematicians, including Pell, Wallis, and possibly Descartes. Her introduction also contains an outline of the six sections of the reassembled *Treatise on Equations* that proves to be very helpful in reading the actual treatise as transcribed on the following 250 pages.

The *Treatise* itself is carefully and accurately transcribed. Other than running titles, Harriot used few words and Stedall has translated these from the Latin. Instead of verbal description, Harriot relied on his symbolic notation and on his arrangement of the equations on the page to convey his meaning. His manuscript work is very clear, and Stedall retains this clarity by preserving as much as possible of both his notation and layout and by

[4] Stedall has since published a complete translation of Wallis's *Arithmetica Infinitorum* as *The Arithmetic of Infinitesimals* (Berlin: Springer-Verlag, 2004).

providing occasional commentary in endnotes. Manuscript volume and folio numbers are given clearly but unobtrusively.

The book contains 13 full-page illustrations, including 11 of Harriot's manuscript sheets, and these give the reader a good sense of what it would be like to read the manuscripts themselves. Of course, transcriptions can be read more quickly and easily, and Stedall has provided a great service to Harriot scholars and to historians of algebra in reassembling and transcribing Harriot's *Treatise on Equations* for us. Her appendix on correlations between the work of Harriot and Viète, between Harriot's work and Torporley's notes, and between Harriot's manuscripts and his published work in the *Praxis* should also be very helpful to scholars. Harriot's algebraic notation and ideas were beautiful, clear, and groundbreaking, as any historian or mathematician who views his work in Stedall's *The Greate Invention of Algebra* will now finally be able to appreciate.

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Essays on the History of Mechanics: In Memory of Clifford Ambrose Truesdell and Edoardo Benvenuto
Edited by Antonio Becchi, Massimo Corradi, Frederico Foce and Orietta Pedemonte. Boston/Basel (Birkhäuser Verlag). 2003. ISBN 3764314761. 256 pp.

This volume of essays was the product of a conference held in Genoa in late 2001 to honor the memory of the eminent historians of mechanics Edoardo Benvenuto (1940–1998) and Clifford Truesdell (1919–2000). Truesdell was a master of the history of theoretical mechanics in the 18th century, while Benvenuto was the author of major works exploring the history of mechanics, civil engineering, and architecture. The essays are divided into three groups. The first group deals with theoretical mechanics with an emphasis on the themes and writings of Truesdell. Included here are essays by Jacques Heyman (theory of structures), Gleb Mikhailov (elasticity theory and structural mechanics), Sandro Caparrini (vectors), Giulio Maltese (Newtonian principles), and Piero Villaggio (impact theories). The second group recalls the work of Benvenuto on civil engineering and architecture, with survey articles by Karl-Eugen Kurrer (deformation method), Santiago Huerta (timbrel vaults), and Patricia Radelet-de-Grave (forces and vaults). Jacques Heyman provides an expository engineering analysis of large glazed windows found in late Romanesque and early Gothic churches. The third group consists of two historiographical articles, a piece by Louis L. Bucciarelli on evaluating error in past mechanics and an essay by David Speiser on history of science and history of fine art. For reasons of space I will discuss only two articles here: Caparrini's essay on the early history of vectors and Bucciarelli's essay on error in past mechanics.

Caparrini's essay won the Slade Prize from the British Society for the History of Science for 2004, a prize awarded biennially to the writer of an essay that makes a critical contribution to the history of science. It is a sequel to a study published in 2002 in the *Archive for History of Exact Science* on the discovery of the vector nature of angular velocity and moments. Caparrini showed that the vector concept emerged in research on the dynamics of rigid bodies between 1760 and 1835. In the present essay he looks at the work of the Italian mathematician Gaetano Giorgini (1795–1874) and the eminent French geometer Michel Chasles (1793–1880). According to standard accounts (e.g., [Crowe, 1967]) in the history of mathematics, the vector concept developed from attempts to represent complex numbers geometrically and from generalizations of these results by William Rowan Hamilton (1805–1865) and Hermann Grassmann (1809–1877). In this view modern vector calculus developed in the work of such physicists as Josiah Willard Gibbs